Supersingular Isogeny Key Encapsulation

Presented by David Jao

University of Waterloo and evolutionQ, Inc.

Full list of submitters:

Reza Azarderakhsh, FAU
Matt Campagna, Amazon
Craig Costello, MSR
Luca De Feo, UVSQ
Basil Hess, ISG
Amir Jalali, FAU
David Jao, UW
Brian Koziel, TI
Brian LaMacchia, MSR
Patrick Longa, MSR
Michael Naehrig, MSR
Joost Renes, Radboud
Vladimir Soukharev, ISG
David Urbanik, UW

http://sike.org

April 11, 2018

SIKE

Supersingular **I**sogeny **K**ey **E**ncapsulation (SIKE)

- IND-CCA2 KEM
- Based on **S**upersingular **I**sogeny **D**iffie-**H**ellman (SIDH)
- Uses Hofheinz et al. transformation (TCC 2017) on SIDH to achieve CCA security

The SIKE protocol specifies:

- Parameter sets
- Key/ciphertext formats
- Encapsulation/decapsulation mechanisms
- Choice of symmetric primitives (hash functions, etc.)

A brief history of SIDH

Couveignes, *Hard Homogeneous Spaces* (1996), ePrint:2006/291

- First explicit mention of isogenies in cryptography
- Unpublished until 2006

Galbraith, *Constructing isogenies between elliptic curves over finite fields* (1999)

- First published cryptanalysis of isogeny problem

Jao and Venkatesan, *Use of isogenies for design of cryptosystems* (2003), US 7499544 (assignee: Microsoft Corporation)

- First (only?) patent on isogeny-based cryptography
- Does not apply to SIDH
- SIDH/SIKE is, to our knowledge, patent-free

Charles et al., *Cryptographic hash functions from expander graphs* (2009)

- First use of supersingular isogenies in cryptography

A brief history of SIDH

Stolbunov, *Constructing public-key cryptographic schemes based on class group action on a set of isogenous elliptic curves* (2010)

- First published isogeny-based public-key cryptosystem
- Essentially identical to Couveignes' unpublished 1996 work
- Partially broken by Childs, Jao, and Soukharev (2014)

Jao and De Feo, *Towards quantum-resistant cryptosystems from supersingular elliptic curve isogenies* (2011)

- Invention of SIDH
- First supersingular isogeny-based public-key cryptosystem

Galbraith et al., *On the Security of Supersingular Isogeny Cryptosystems* (2016)

- Active attack against SIDH with static key re-use
- Necessitates use of Hofheinz et al. transform for CCA security

Overview of SIDH

1. Public parameters: Supersingular elliptic curve E over F.
2. Alice chooses a kernel $A \subset E$ and sends E/A to Bob.
3. Bob chooses a kernel $B \subset E$ and sends E/B to Alice.
4. The shared secret is

$$E/\langle A, B\rangle = (E/A)/\phi_A(B) = (E/B)/\phi_B(A).$$

$$\begin{tikzcd} E \arrow[r, "\phi_A"] \arrow[d, "\phi_B"'] & E/A \arrow[d] \\ E/B \arrow[r] & E/\langle A, B\rangle \end{tikzcd}$$

Detailed description of SIDH

Public parameters:

- Prime $p = 2^{e_2}3^{e_3} - 1$
- Supersingular elliptic curve $E/\mathbb{F}_{p^2}$ of order $(p+1)^2$
- $\mathbb{Z}$-basis $\{P_2, Q_2\}$ of $E[2^{e_2}]$ and $\{P_3, Q_3\}$ of $E[3^{e_3}]$

Alice:

- Choose $\mathrm{sk}_2 \in \mathbb{Z}$ and compute $S_2 = P_2 + \mathrm{sk}_2 Q_2$ of order 2^{e_2}
- Compute $\phi_2 \colon E \to E/\langle S_2 \rangle$
- Send $E/\langle S_2 \rangle, \phi_2(P_3), \phi_2(Q_3)$ to Bob

Bob:

- Same as Alice, swapping 2 with 3

The shared secret is derived from

$$\begin{aligned} E/\langle S_2, S_3 \rangle &= (E/\langle S_2 \rangle)/\langle \phi_2(P_3) + \mathrm{sk}_3 \phi_2(Q_3) \rangle \\ &= (E/\langle S_3 \rangle)/\langle \phi_3(P_2) + \mathrm{sk}_2 \phi_3(Q_2) \rangle \end{aligned}$$

SIKE parameter sets

`SIKEp503`:

- $p = 2^{250}3^{159} - 1$ (note, the value of this prime is listed incorrectly in the spec)
- $P_2 = 3^{159} \cdot E(i+4),\ Q_2 = 3^{159} \cdot E(14)$
- $P_3 = 2^{250} \cdot E(i+7),\ Q_3 = 2^{250} \cdot E(6)$

`SIKEp751`:

- $p = 2^{372}3^{239} - 1$
- $P_2 = 3^{239} \cdot E(i+5),\ Q_2 = 3^{239} \cdot E(11)$
- $P_3 = 2^{372} \cdot E(i+2),\ Q_3 = 2^{372} \cdot E(6)$

`SIKEp964`:

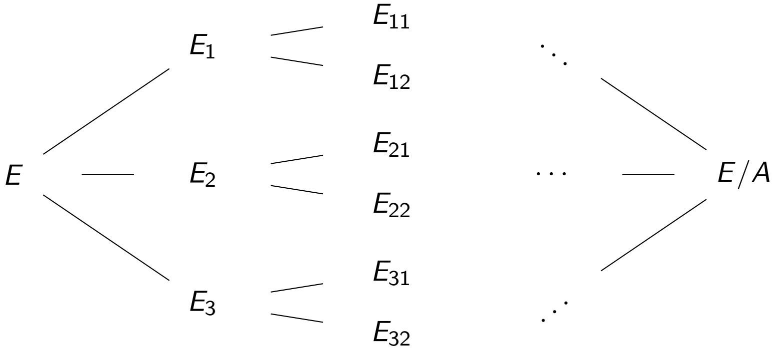
- $p = 2^{486}3^{301} - 1$
- $P_2 = 3^{301} \cdot E(i+23),\ Q_2 = 3^{301} \cdot E(11)$
- $P_3 = 2^{486} \cdot E(i+1),\ Q_3 = 2^{486} \cdot E(5)$

N.b.: $i = \sqrt{-1} \in \mathbb{F}_{p^2}$, $E : y^2 = x^3 + x$ and $E(x) = (x, \sqrt{x^3 + x})$.

Attack complexity

Hardness problem: Given E and E/A with a guarantee of the existence of $\phi\colon E \to E/A$, find A.

Fastest known (passive) attack is a generic collision search or claw search on a space of size $\deg(\phi)$:



Security

In principle, a non-generic attack against SIKE could conceivably exist; however, none is currently known. For **generic** attacks:

parameter set	security	NIST category
`SIKEp503`	`SHA256`	2
`SIKEp751`	`SHA384`	4
`SIKEp964`	`AES256/SHA512`	5

Recent developments pertaining to SIDH/SIKE security:

- Petit (Asiacrypt 2017): non-generic attacks against "unbalanced" versions of SIDH (**not used** in SIKE)
- Petit and Lauter, ePrint 2017/962: reductions from the isogeny problem to finding supersingular endomorphism rings
- Urbanik and Jao, AsiaPKC 2018: random self-reducibility
- Adj et al., ePrint:2018/313: proposes smaller parameters for 128-bit security, based on more detailed analysis of attacks

Implementation

```
        0.069188618 s   KEX Total   [FrodoKEM-640]
        0.075546943 s   KEX Total   [NTS-KEM(13, 80)]
        0.114103121 s   KEX Total   [Ramstake RS 756839]
        0.117327944 s   KEX Total   [ODD_MANHATTAN]
        0.127024638 s   KEX Total   [RLCEKEM128B]
        0.136131757 s   KEX Total   [DME-KEM (N=2, M=3, E=48, S=3)]
        0.148760336 s   KEX Total   [NTS-KEM(13, 136)]
        0.152088446 s   KEX Total   [FrodoKEM-976]
        0.190694193 s   KEX Total   [SIKEp503]
        0.646993100 s   KEX Total   [SIKEp751]
        0.683500220 s   KEX Total   [CFPKM-128]
        1.009693669 s   KEX Total   [Classic McEliece 8192128$]
        1.214073736 s   KEX Total   [BIG_QUAKE_1]
        1.679732008 s   KEX Total   [Classic McEliece 6960119]
        2.033252376 s   KEX Total   [CFPKM-182]
        2.334988284 s   KEX Total   [Post-Quantum RSA Enc - pqrsa15]
        4.365430313 s   KEX Total   [BIG_QUAKE_3]
        7.288352877 s   KEX Total   [DAGS_3]
        8.105539551 s   KEX Total   [BIG_QUAKE]
       52.913978368 s   KEX Total   [DAGS_5]
```

(credit: pqbench by Markku-Juhani O. Saarinen)

Key sizes:

- SIKEp503 — 378 bytes
- SIKEp751 — 564 bytes
- SIKEp964 — 726 bytes

- Performance with platform-specific Intel64 assembly optimizations (AVX2) is $\sim$ 9x faster
- Key compression (Zanon et al., PQCrypto 2018):
  - $\sim$ 40% smaller keys
  - $\sim$ 2x slower performance
  - Not included in SIKE specification, for the sake of simplicity

Summary

SIKE advantages:

- Very small key sizes
- No possibility for decryption error
- No complicated error distributions, rejection sampling, etc.
- Simple, conservative security analysis when assuming only generic attacks

SIKE disadvantages:

- Relatively slow
- Future analysis may uncover non-generic attacks against SIKE (though none are known so far)